PROBLEMS on Convergence of Improper Integrals.

1) Test the Convergence of $\int_0^\infty e^{-x} dx$

We have $\int_0^\infty e^{-x} dx = \lim_{x\to\infty} \int_0^x e^{-x} dx$

$$= \lim_{x\to\infty} \left[\frac{e^{-x}}{-1} \right]_0^x$$

$$= \lim_{x\to\infty} -1 \left[e^{-x} - e^{-0} \right]$$

$$= \lim_{x\to\infty} \left[1 - e^{-x} \right]$$

$$= 1 \qquad \text{Since } \left[e^{-\infty} = 0 \right]$$

Thus the limit exists and unique and finite and hence the integral is cgt.

(2) Test the Convergence of $\int_0^\infty \cos x \, dx$

$$= \lim_{x\to\infty} \int_0^x \cos x \, dx$$

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$$= \lim_{x\to\infty}\left[\sin x\right]_0^x$$

The above limit does not exist as a unique value since the limit of $\sin x$ oscillates between $+1$ and -1 when $x \to \infty$. Hence the given integral oscillates.

(3) Test the Convergence of

$$\int_{-\infty}^{0} e^{-x}\,dx$$

$$\lim_{x\to-\infty}\int_x^0 e^{-x}\,dx$$

$$= \lim_{x\to-\infty}\left[-e^{-x}\right]_x^0 = \lim_{x\to-\infty}\left[1 - e^{-x}\right]$$

$$= \lim_{x\to-\infty}\left[e^{-x} - 1\right]$$

$$= \infty$$

Hence the integral diverges to $+\infty$.

(4) Test the Convergence of $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}\,dx$

Solⁿ $$\int_{-\infty}^{\infty}\frac{dx}{1+x^2} = \int_{-\infty}^{0}\frac{dx}{1+x^2} + \int_0^{\infty}\frac{dx}{1+x^2}\,dx$$

Expt. No.............. Date .... Page No. ....

$$= \lim_{x\to-\infty}\int_x^0 \frac{dx}{1+x^2} + \lim_{x\to\infty}\int_0^x \frac{dx}{1+x^2}$$

$$= \lim_{x\to-\infty}\left[\tan^{-1}x\right]_x^0 + \lim_{x\to\infty}\left[\tan^{-1}x\right]_0^x$$

$$= \lim_{x\to-\infty}\left[0 - \tan^{-1}x\right] + \lim_{x\to\infty}\left[\tan^{-1}x - 0\right]$$

$$= \lim_{x\to-\infty}\left[-\tan^{-1}x\right] + \lim_{x\to\infty}\tan^{-1}x$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

Hence the given integral Converges.

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